OGAP Engineering Questions and Problem Structures

About this Page: The problems below illustrate how OGAP items are engineered to elicit specific evidence in relationship to both a *mathematical goal* and in response to *math education research* by intentionally altering problem structures. (These examples do not represent all the ways questions can be modified to respond to this math goal or the research.)

Mathematical Goal: Students use reasoning strategies to compare fractions.

Key math education research: Behr & Post indicate that "a child's understanding of the ordering of two fractions needs to be based on an understanding of the ordering of unit fractions" (1992, p.21)

Q1) Put the following fractions in order from smallest to largest. Explain your thinking.

$1/8 \quad 1/125 \quad 1/13 \quad 1/57$

About Q1: Q1 involves ordering 4 unit fractions with a range of magnitudes. Because of the magnitudes of the unit fractions, Q1 has the potential to elicit:

- Unit fraction reasoning
- Inappropriate whole number reasoning.

Strategies such as using visual models or common denominators are not reasonable given the relative magnitude of the fractions.

Q2) Sheila believes that the inequality below is a true statement. Is she correct or incorrect? Explain your reasoning.

$$\frac{1}{5}+\frac{1}{5}+\frac{1}{5} > \frac{1}{4}+\frac{1}{4}+\frac{1}{4}$$

About Q2: Q2 involves comparing familiar unit fractions in the context of adding unit fractions. It has the potential to elicit:

- Understanding of addition of unit fractions
- Use of unit fraction reasoning (e.g., fifths are smaller than fourths so three-fifths is less than three-fourths. Sheila is incorrect.)
- Use of visual model
- Possibly, inappropriate whole number reasoning (e.g., 15>12 so Sheila is correct.)

Q3) Which fraction is closer to 1/2? Explain your thinking.

$$\frac{3}{4} \text{ or } \frac{5}{12}$$

About Q3: Q3 involves comparing fractions with different denominators and different numerators to a benchmark. Because 3/4 is 1/4 greater than 1/2 and 5/12 is 1/12 less than 1/2, the problem has the potential to elicit:

- Benchmark reasoning
- Use of extended unit fraction reasoning
- Use of visual model
- Use of common denominators
- Possibly, inappropriate whole number reasoning

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OGAP Fraction Framework

Depending upon the strength of fractional reasoning students may move up and down between fractional, transitional, early transition, and non-fractional reasoning and strategies as they interact with new topics or new concepts (Petit, Laird, & Marsden (in press 2015)).

Fraction Problem Structures

Fraction Topics
Partitioning
Compare and order
Equivalence
Number lines
Operations
Density of fractions

Partitioning
Algorithmic halving
Oddness
Evenness
Composition

Fraction Types
Unit fractions
Non-unit fractions
Proper fractions
Improper fractions
Mixed numbers
Negative fractions

Visual Models
To solve problems
To understand concepts
To generalize concepts
Area
Sets
Number line

Number Lines
0-1
Negative to positive
Two or more units
Unpartitioned
Partitioned

Wholes
Same size
Different size
Given part, find whole

Number of Parts in Whole
Equal to denominator
Multiples of the denominator
Factors of the denominators

Reasoning Strategies
Number sense
Unit fraction
Extended unit fraction
Using visual models
Benchmark
Equivalence
Properties of operations

Class of Fractions
Different numerators, same denominators
Same numerators, different denominators
Different numerators, different denominators

Operations
All operations
Estimation
Efficient algorithm
Impact of operation
Equivalence

Division
Partitive division
Quotative division

Types of Problems
Requires interaction with a visual model
An exact numerical answer is required
An exact numerical answer is NOT required
Contextual
Non-contextual
Supporting a claim
Mathematical explanation required
Multi-step problems
Link equation to a contextual problem
Link equation to visual model
Extended multiple choice
Impact of operations

About OGAP Fraction Framework

The OGAP Fraction Framework is based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

There are three major elements to the OGAP Fraction Framework that should be considered when analyzing student work or making instructional decisions:

1. Problem structures
2. Evidence in student work along a progression
3. Evidence of underlying issues or errors

This page identifies problem structures for fraction problems that can be used in planning a lesson or selecting or designing a task. The centerfold is a learning progression designed to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions and provide feedback to students. The back page contains some examples of engineered questions based on mathematics education research.

Consistent with the CCSSM the OGAP Fraction Progression uses visual models, equipartitioning, unit fraction understanding, equivalence, and properties of operations as means to developing understanding and fluency of fractions. Ultimately, fluency will enable students to engage in middle school topics that assume proficiency with fractions (e.g., proportionality, solving equations with fractional coefficients).

As students interact with new concepts, new structures, and more complex problem solving situations they may move back and forth between fractional, transitional, early fractional, and non-fractional reasoning and strategies. This is important evidence to use for instructional decision-making. For example, a student may consistently find a fractional part of a set or area by physically partitioning a given visual model. However, when asked to find 3/4 of 164, students may revert to a non-fractional strategy.

To learn more about the mathematics education research underpinning the OGAP Fraction Framework read *A Focus on Fractions: Bringing Research to the Classroom* (Petit, Laird, & Marsden). OGAP references are found at **www.ogapmath.com**.

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OGAP Fraction Progression

Note: The examples provided do NOT represent the full set of possible solutions that represent each level.

As students learn new concepts or interact with new structures or problem situations they may move back and forth across these levels.

Application

Middle school topics and concepts in which rational number understanding and procedures are applied:

Area, Volume, Surface Area	Proportions	Similarity	Probability
Percents	Rates	Transformations	Functions
Expressions and equations	Scaling	Measures of Central Tendency	Others

Fractional Strategies

Accurately locates fractions on a number line of any length, compares and orders fractions using a range of strategies, finds equivalent fractions, and operates efficiently when solving mathematical and contextual problems

- Uses reasoning about relative magnitude
- Uses benchmark reasoning
- Uses unit fraction reasoning
- Uses equivalence
- Uses efficient algorithm
- Uses properties of operations
- Demonstrates understanding of concept
- Equipartitions a given visual model

Distributive property

Bob ran 4 times this week. Each run was 3 3/4 miles long. How far did Bob run this week?

$4 \times 3\frac{3}{4} = 12\frac{12}{4}$

or 15 miles

Unit fraction and benchmark reasoning

Which fraction is closest to 1?

$\frac{7}{3}$ $\frac{7}{5}$ $\frac{7}{6}$ $\frac{7}{12}$

7/6 is 1/6 away from 1

7/12 is 5/12 away from one, 5/12 is larger then 1/6

7/5 is 2/5 larger then 1, 2/5 is larger then 1/6

7/3 = 2 1/3

Magnitude Reasoning

Aunt Sally has a jar that holds one cup of liquid. Her salad dressing recipe calls for 2/3 cup of oil, 1/8 cup of vinegar, and 1/4 cup of juice. Is the jar large enough to hold the oil, vinegar, and juice?

The jar is not large enough to hold all the oil, vinegar, and lemon

1/8 + 1/4 = 3/8

3/8 is larger than 1/3.

I got 1/3 because she used 2/3 of a cup of oil.

Algorithm

Jim is making decorations for a school dance. He has 4 1/4 yards of wire. Each decoration needs 3/4 of a yard of wire. How many full decorations can Jim make?

$4\frac{1}{4} = \frac{17}{4}$

5 full decorations

$\frac{17}{4} \times \frac{4}{3} = \frac{68}{12} = 5\frac{8}{12} = 5\frac{2}{3}$

Transitional Strategies

Effectively generates a visual model to solve problems and show understanding.

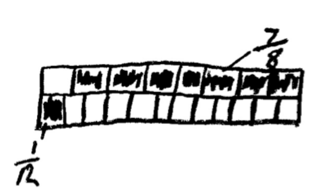
The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:

a) 20

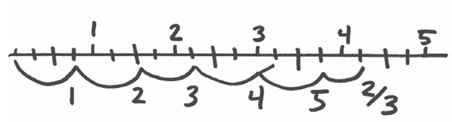
b) 8

c) $\frac{1}{2}$

d) 1



Jim is making decorations for a school dance. He has 4 1/4 yards of wire. Each decoration needs 3/4 of a yard of wire. How many full decorations can Jim make?



Jim can make 5 full decorations

Evidence of fractional thinking, but reasoning or strategy is not efficient

Bob ran 4 times this week. Each run was 3 3/4 miles long. How far did Bob run this week?

3 3/4
3 3/4
3 3/4
+ 3 3/4
12 12/4 = 15

Uses Unit Fraction Strategy

A fruit punch recipe calls for 2 2/3 cups of lemonade. How many times would Jim need to fill a 1/3 cup to measure the correct amount of lemonade to put in the fruit punch?

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

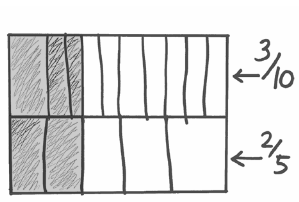
$1\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 2$

$2 + \frac{1}{3} + \frac{1}{3} = 2\frac{2}{3}$

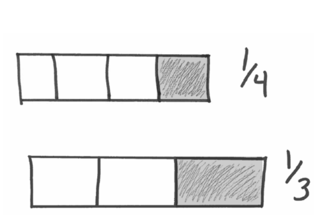
8

Early Fractional Strategies

Uses a fractional or transitional strategy (like partitioning visual models) or an operation appropriate for the situation, but the solution includes an error (e.g., partitioning, size of whole, concept error in part of problem)



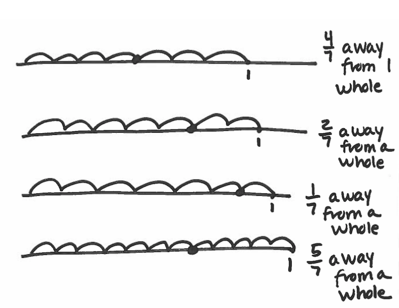
They are equal.



Which fraction is closest to 1?

$\frac{7}{3}$ $\frac{7}{5}$ $\frac{7}{6}$ $\frac{7}{12}$

7/6 is bigger because it is 1/7 away from a whole



Non-Fractional

Applies rules without evidence of understanding, inappropriate whole number reasoning, or uses an incorrect operation given the problem context

Which fraction is closest to 1?

$\frac{7}{3}$ $\frac{7}{5}$ $\frac{7}{6}$ $\frac{7}{12}$

$\underbrace{\frac{7}{3}\ \frac{7}{5}\ \frac{7}{6}}_{\text{All are big-headed}}$ $\frac{7}{12}$ NOT big-headed

Stephanie and Paige are discussing the answer to 3 2/7 x 5/9. Stephanie said that the answer is more than 3 2/7. Paige said the answer is less than 3 2/7. Who is correct?

If you multiply anything it has to be bigger than what you multiply by. Stephanie is right.

Whole number reasoning, not fractional reasoning

The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:

a) 20 b) 8 c) $\frac{1}{2}$ d) 1

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20

Incorrect operation given context

Jim is making decorations for a school dance. He has 4 1/4 yards of wire. Each decoration needs 3/4 of a yard of wire. How many full decorations can Jim make?

$\frac{17}{4} \times \frac{3}{4} = \frac{51}{16}$

$3 \times 16 = 48$

3 3/16 decorations

Use of visual models, equipartitioning, unit fraction understanding, equivalence, and properties of operations

Underlying Issues/Errors

Rule, without understanding	Inappropriate whole number reasoning	Errors in: Size of whole	Concept	Partitioning	Property	Remainder
Misinterprets model	Incorrect operation		Calculation	Procedure	Equation	Other

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