CHAPTER 7 TRANSPOSITION OF FORMULAE

EXERCISE 30, Page 60

1. Make d the subject of the formula: \[ a + b = c - d - e \]

Since \[ a + b = c - d - e \] then \[ d = c - e - a - b \]

2. Make x the subject of the formula: \[ y = 7x \]

Dividing both sides of \[ y = 7x \] by 7 gives: \[ x = \frac{y}{7} \]

3. Make v the subject of the formula: \[ pv = c \]

Dividing both sides of \[ pv = c \] by p gives: \[ v = \frac{c}{p} \]

4. Make ‘a’ the subject of the formula: \[ v = u + at \]

Since \[ v = u + at \] then \[ v - u = at \]

and dividing both sides by t gives: \[ \frac{v - u}{t} = a \] or \[ a = \frac{v - u}{t} \]

5. Make y the subject of the formula: \[ x + 3y = t \]

Since \[ x + 3y = t \] then \[ 3y = t - x \]

and dividing both sides by 3 gives: \[ y = \frac{t - x}{3} \] or \[ y = \frac{1}{3}(t - x) \]

6. Make r the subject of the formula: \[ c = 2\pi r \]
Dividing both sides of \( c = 2\pi r \) by \( 2\pi \) gives: \( \frac{c}{2\pi} = r \) or \( r = \frac{c}{2\pi} \)

7. Make \( x \) the subject of the formula: \( y = mx + c \)

Since \( y = mx + c \) then \( y - c = mx \)

and dividing both sides by \( m \) gives: \( \frac{y - c}{m} = x \) or \( x = \frac{y - c}{m} \)

8. Make \( T \) the subject of the formula: \( I = PRT \)

Dividing both sides of \( I = PRT \) by \( PR \) gives: \( \frac{I}{PR} = T \) or \( T = \frac{I}{PR} \)

9. Make \( L \) the subject of the formula: \( X_L = 2\pi f L \)

Dividing both sides of \( X_L = 2\pi f L \) by \( 2\pi f \) gives: \( \frac{X_L}{2\pi f} = L \) or \( L = \frac{X_L}{2\pi f} \)

10. Make \( R \) the subject of the formula: \( I = \frac{E}{R} \)

Multiplying both sides of \( I = \frac{E}{R} \) by \( R \) gives: \( IR = E \)

and dividing both sides by \( I \) gives: \( R = \frac{E}{I} \)

11. Make \( x \) the subject of the formula: \( y = \frac{x}{a} + 3 \)

Since \( y = \frac{x}{a} + 3 \) then \( y - 3 = \frac{x}{a} \)

Multiplying both sides by ‘a’ gives: \( a(y - 3) = x \) or \( x = a(y - 3) \)
12. Make C the subject of the formula: \( F = \frac{9}{5}C + 32 \)

Rearranging \( F = \frac{9}{5}C + 32 \) gives: \( F - 32 = \frac{9}{5}C \)

Multiplying both sides by \( \frac{5}{9} \) gives: 
\[
\frac{5}{9} (F - 32) = \left( \frac{5}{9} \right) \left( \frac{9}{5}C \right)
\]

i.e. 
\[
\frac{5}{9} (F - 32) = C \quad \text{or} \quad C = \frac{5}{9} (F - 32)
\]
EXERCISE 31, Page 61

1. Make \( r \) the subject of the formula: \( S = \frac{a}{1-r} \)

Multiplying both sides of \( S = \frac{a}{1-r} \) by \((1-r)\) gives: \( S(1-r) = a \)

i.e. \( S - Sr = a \)

from which,

\( S - a = Sr \)

and dividing both sides by \( S \) gives:

\( \frac{S-a}{S} = r \) i.e. \( r = \frac{S-a}{S} \) or \( r = 1 - \frac{a}{S} \)

2. Make \( x \) the subject of the formula: \( y = \frac{\lambda(x-d)}{d} \)

Multiplying both sides of \( y = \frac{\lambda(x-d)}{d} \) by \( d \) gives: \( yd = \lambda(x-d) \)

Dividing both sides by \( \lambda \) gives:

\( \frac{yd}{\lambda} = x - d \)

and

\( d + \frac{yd}{\lambda} = x \) or \( x = d + \frac{yd}{\lambda} \)

Alternatively, from the first step, \( yd = \lambda(x-d) \)

i.e. \( yd = \lambda x - \lambda d \)

and \( yd + \lambda d = \lambda x \)

from which,

\( x = \frac{yd + \lambda d}{\lambda} = \frac{d(y + \lambda)}{\lambda} \) i.e. \( x = \frac{d}{\lambda}(y + \lambda) \)

3. Make \( f \) the subject of the formula: \( A = \frac{3(F-f)}{L} \)
Multiplying both sides of \( A = \frac{3(F-f)}{L} \) by \( L \) gives: \( AL = 3(F-f) \)

Dividing both sides by 3 gives: \( \frac{AL}{3} = F-f \)

and \( f = F - \frac{AL}{3} \) or \( f = \frac{3F - AL}{3} \)

4. Make \( D \) the subject of the formula: \( y = \frac{AB^2}{5CD} \)

Multiplying both sides of \( y = \frac{AB^2}{5CD} \) by \( D \) gives: \( yD = \frac{AB^2}{5C} \)

Dividing both sides by \( y \) gives: \( D = \frac{AB^2}{5Cy} \)

5. Make \( t \) the subject of the formula: \( R = R_0(1 + \alpha t) \)

Removing the bracket in \( R = R_0(1 + \alpha t) \) gives: \( R = R_0 + R_0 \alpha t \)

from which, \( R - R_0 = R_0 \alpha t \)

and \( \frac{R - R_0}{R_0 \alpha} = t \) or \( t = \frac{R - R_0}{R_0 \alpha} \)

6. Make \( R_2 \) the subject of the formula: \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \)

Rearranging \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \) gives: \( \frac{1}{R} - \frac{1}{R_1} = \frac{1}{R_2} \)

i.e. \( \frac{1}{R_2} = \frac{1}{R} - \frac{1}{R_1} = \frac{R_1 - R}{RR_1} \)

Turning both sides upside down gives: \( R_2 = \frac{RR_1}{R_1 - R} \)
7. Make R the subject of the formula: \[ I = \frac{E - e}{R + r} \]

Multiplying both sides by \((R + r)\) gives: \[ I(R + r) = E - e \]

i.e. \[ IR + Ir = E - e \] and \[ IR = E - e - Ir \]

and dividing both sides by \(I\) gives: \[ R = \frac{E - e - Ir}{I} \quad \text{or} \quad R = \frac{E - e}{I} - r \]

8. Make \(b\) the subject of the formula: \[ y = 4ab^2c^2 \]

Dividing both sides by \(4ac^2\) gives: \[ \frac{y}{4ac^2} = b^2 \quad \text{or} \quad b^2 = \frac{y}{4ac^2} \]

Taking the square root of both sides gives: \[ b = \sqrt[2]{\frac{y}{4ac^2}} \]

9. Make \(x\) the subject of the formula: \[ \frac{a^2}{x^2} + \frac{b^2}{y^2} = 1 \]

Rearranging \(\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1\) gives: \[ \frac{a^2}{x^2} = 1 - \frac{b^2}{y^2} = \frac{y^2 - b^2}{y^2} \]

Turning both sides upside down gives: \[ \frac{x^2}{a^2} = \frac{y^2}{y^2 - b^2} \]

Multiplying both sides by \(a^2\) gives: \[ x^2 = a^2 \left( \frac{y^2}{y^2 - b^2} \right) = \frac{a^2y^2}{y^2 - b^2} \]

Taking the square root of both sides gives: \[ x = \sqrt[2]{\frac{a^2y^2}{y^2 - b^2}} = \frac{\sqrt[2]{a^2y^2}}{\sqrt[2]{y^2 - b^2}} = \frac{ay}{\sqrt[2]{y^2 - b^2}} \]

i.e. \[ x = \frac{ay}{\sqrt[2]{y^2 - b^2}} \]
10. Make \( L \) the subject of the formula:  
\[ t = 2\pi \frac{L}{\sqrt{g}} \]

Dividing both sides of \( t = 2\pi \frac{L}{\sqrt{g}} \) by \( 2\pi \) gives:  
\[ \frac{t}{2\pi} = \frac{L}{\sqrt{g}} \]

Squaring both sides gives:  
\[ \left( \frac{t}{2\pi} \right)^2 = \frac{L}{g} \quad \text{or} \quad \frac{L}{g} = \left( \frac{t}{2\pi} \right)^2 \]

Multiplying both sides by \( g \) gives:  
\[ L = g\left( \frac{t}{2\pi} \right)^2 \quad \text{or} \quad L = \frac{gt^2}{4\pi^2} \]

11. Make \( u \) the subject of the formula:  
\[ v^2 = u^2 + 2as \]

Since \( v^2 = u^2 + 2as \) then \( v^2 - 2as = u^2 \) or \( u^2 = v^2 - 2as \)

Taking the square root of each side gives:  
\[ u = \sqrt{v^2 - 2as} \]

12. Make ‘\( a \)’ the subject of the formula:  
\[ N = \sqrt{\frac{a+x}{y}} \]

Squaring both sides of \( N = \sqrt{\frac{a+x}{y}} \) gives:  
\[ N^2 = \frac{a+x}{y} \]

Multiplying both sides by \( y \) gives:  
\[ N^2y = a + x \quad \text{or} \quad a + x = N^2y \]

from which,  
\[ a = N^2y - x \]

13. The lift force, \( L \), on an aircraft is given by:  
\[ L = \frac{1}{2} \rho v^2 a c \]

where \( \rho \) is the density, \( v \) is the velocity, \( a \) is the area and \( c \) is the lift coefficient. Transpose the equation to make the velocity the subject.
Since \( L = \frac{1}{2} \rho v^2 a c \) then \( \frac{2L}{\rho ac} = v^2 \)

from which, velocity, \( v = \sqrt{\frac{2L}{\rho ac}} \)
EXERCISE 32, Page 62

1. Make ‘a’ the subject of the formula: \( y = \frac{a^2m - a^2n}{x} \)

   Multiplying both sides of \( y = \frac{a^2m - a^2n}{x} \) by \( x \) gives: \( xy = a^2(m - n) \)

   and factorising gives: \( xy = a^2(m - n) \)

   Dividing both sides by \( (m - n) \) gives: \( \frac{xy}{m - n} = a^2 \) or \( a^2 = \frac{xy}{m - n} \)

   Taking the square root of both sides gives: \( a = \sqrt{\frac{xy}{m - n}} \)

2. Make \( R \) the subject of the formula: \( M = \pi(R^4 - r^4) \)

   Dividing both sides of \( M = \pi(R^4 - r^4) \) by \( \pi \) gives: \( \frac{M}{\pi} = R^4 - r^4 \)

   and rearranging gives: \( \frac{M}{\pi} + r^4 = R^4 \) or \( R^4 = \frac{M}{\pi} + r^4 \)

   Taking the fourth root of both sides gives: \( R = \sqrt[4]{\frac{M}{\pi} + r^4} \)

3. Make \( r \) the subject of the formula: \( x + y = \frac{r}{3 + r} \)

   Multiplying both sides of \( x + y = \frac{r}{3 + r} \) by \( (3 + r) \) gives: \( (x + y)(3 + r) = r \)

   Multiplying the brackets gives: \( 3x + xr + 3y + yr = r \)

   and rearranging gives: \( xr + yr - r = -3x - 3y \)

   Factorising gives: \( r(x + y - 1) = -3(x + y) \)

   Dividing both sides by \( (x + y - 1) \) gives: \( r = \frac{-3(x + y)}{x + y - 1} \)
Multiplying numerator and denominator by -1 gives:  \( r = \frac{3(x + y)}{1 - x - y} \)

4. Make L the subject of the formula:  \( m = \frac{\mu L}{L + rCR} \)

Multiplying both sides of \( m = \frac{\mu L}{L + rCR} \) by \((L + rCR)\) gives: \( m(L + rCR) = \mu L \)

Removing brackets gives: \( mL + mrCR = \mu L \)

and rearranging gives: \( mrCR = \mu L - mL \)

Factorising gives: \( mrCR = L(\mu - m) \)

Dividing both sides by \((\mu - m)\) gives:

\[ L = \frac{mrCR}{\mu - m} \]

5. Make b the subject of the formula:  \( a^2 = \frac{b^2 - c^2}{b^2} \)

Multiplying both sides by \( b^2 \) gives: \( a^2b^2 = b^2 - c^2 \)

and rearranging gives: \( c^2 = b^2 - a^2b^2 \) or \( b^2 - a^2b^2 = c^2 \)

Factorising gives: \( b^2(1 - a^2) = c^2 \)

Dividing both sides by \((1 - a^2)\) gives: \( b^2 = \frac{c^2}{1 - a^2} \)

Taking the square root of both sides gives: \( b = \sqrt{\frac{c^2}{1 - a^2}} = \frac{\sqrt{c^2}}{\sqrt{1 - a^2}} \)

Hence, \( b = \frac{c}{\sqrt{1 - a^2}} \)

6. Make r the subject of the formula:  \( \frac{x}{y} = \frac{1 + r^2}{1 - r^2} \)
Rearranging by ‘cross-multiplying’ gives: 
\[ x(1-r^2) = y(1+r^2) \]

Removing brackets gives: 
\[ x - x^2 = y + yr^2 \]

and rearranging gives: 
\[ x - y = yr^2 + x^2 \] or \[ yr^2 + x^2 = x - y \]

Factorising gives: 
\[ r^2(x + y) = x - y \]

Dividing both sides by \((x + y)\) gives: 
\[ r^2 = \frac{x - y}{x + y} \]

Taking the square root of both sides gives: 
\[ r = \sqrt{\frac{x - y}{x + y}} \]

7. A formula for the focal length, \(f\), of a convex lens is: 
\[ \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \]. Transpose the formula to make \(v\) the subject and evaluate \(v\) when \(f = 5\) and \(u = 6\)

Rearranging \[ \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \] gives: 
\[ \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u - f}{uf} \]

Turning each side upside down gives: 
\[ v = \frac{uf}{u - f} \]

When \(f = 5\) and \(u = 6\), then 
\[ v = \frac{uf}{u - f} = \frac{(6)(5)}{6 - 5} = \frac{30}{1} = 30 \]

8. The quantity of heat, \(Q\), is given by the formula \(Q = mc(t_2 - t_1)\). Make \(t_2\) the subject of the formula and evaluate \(t_2\) when \(m = 10\), \(t_1 = 15\), \(c = 4\) and \(Q = 1600\)

Removing the brackets in \(Q = mc(t_2 - t_1)\) gives: 
\[ Q = mct_2 - mct_1 \]

and rearranging gives: 
\[ Q + mct_1 = mct_2 \]

or 
\[ mct_2 = Q + mvt_1 \]

Dividing both sides by \(mc\) gives: 
\[ t_2 = \frac{Q + mvt_1}{mc} \] or \[ t_2 = \frac{Q}{mc} + t_1 \] or \[ t_2 = t_1 + \frac{Q}{mc} \]

When \(m = 10\), \(t_1 = 15\), \(c = 4\) and \(Q = 1600\),
9. The velocity, $v$, of water in a pipe appears in the formula $h = \frac{0.03Lv^2}{2dg}$. Express $v$ as the subject of the formula and evaluate $v$ when $h = 0.712$, $L = 150$, $d = 0.30$ and $g = 9.81$

Multiplying both sides of $h = \frac{0.03Lv^2}{2dg}$ by $2dg$ gives: $2dgh = 0.03Lv^2$

Dividing both sides by $0.03L$ gives: $v^2 = \frac{2dgh}{0.03L}$

Taking the square root of each side gives: $v = \sqrt{\frac{2dgh}{0.03L}}$

When $h = 0.712$, $L = 150$, $d = 0.30$ and $g = 9.81$,

$$v = \sqrt{\frac{2dgh}{0.03L}} = \sqrt{\frac{2(0.30)(9.81)(0.712)}{0.03(150)}} = \sqrt{0.931296} = 0.965$$

10. The sag $S$ at the centre of a wire is given by the formula: $S = \sqrt{\frac{3d(l-d)}{8}}$

Make $l$ the subject of the formula and evaluate $l$ when $d = 1.75$ and $S = 0.80$

Squaring both sides of $S = \sqrt{\frac{3d(l-d)}{8}}$ gives: $S^2 = \frac{3d(l-d)}{8}$

Multiplying both sides by 8 gives: $8S^2 = 3d(l-d)$

Removing the bracket gives: $8S^2 = 3dl - 3d^2$

Rearranging gives: $8S^2 + 3d^2 = 3dl$

or $3dl = 8S^2 + 3d^2$

Dividing both sides by 3d gives: $l = \frac{8S^2 + 3d^2}{3d} = \frac{8S^2}{3d} + \frac{3d^2}{3d}$
i.e. \[ l = \frac{8S^2}{3d} + d \]

When \( d = 1.75 \) and \( S = 0.80 \), \[ l = \frac{8(0.80)^2}{3(1.75)} + 1.75 = 0.975 + 1.75 = 2.725 \]

**11.** An approximate relationship between the number of teeth, \( T \), on a milling cutter, the diameter of cutter, \( D \), and the depth of cut, \( d \), is given by: \[ T = \frac{12.5D}{D + 4d} \]

Determine the value of \( D \) when \( T = 10 \) and \( d = 4 \) mm.

Multiplying both sides of \[ T = \frac{12.5D}{D + 4d} \] by \( D + 4d \) gives: \[ T(D + 4d) = 12.5D \]

Removing brackets gives: \[ TD + 4dT = 12.5D \]

Rearranging gives: \[ 4dT = 12.5D – TD \]

or \[ 12.5D – TD = 4dT \]

Factorising gives: \[ D(12.5 – T) = 4dT \]

Dividing both sides by \( (12.5 – T) \) gives: \[ D = \frac{4dT}{12.5 – T} \]

When \( T = 10 \) and \( d = 4 \) mm, then \[ D = \frac{4(4)(10)}{12.5 – 10} = \frac{160}{2.5} = 64 \text{ mm} \]

**12.** A simply supported beam of length \( L \) has a centrally applied load \( F \) and a uniformly distributed load of \( w \) per metre length of beam. The reaction at the beam support is given by:

\[ R = \frac{1}{2}(F + wL) \]

Rearrange the equation to make \( w \) the subject. Hence determine the value of \( w \) when \( L = 4 \) m, \( F = 8 \) kN and \( R = 10 \) kN

Since \[ R = \frac{1}{2}(F + wL) \] then \[ 2R = F + wL \]

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and \[2R - F = wL\]

from which, \[w = \frac{2R - F}{L}\]

When \(L = 4\) m, \(F = 8\) kN and \(R = 10\) kN, \(w = \frac{2(10) - 8}{4} = \frac{12}{4} = 3\) kN/m

13. The rate of heat conduction through a slab of material, \(Q\), is given by the formula

\[Q = \frac{kA(t_1 - t_2)}{d}\]

where \(t_1\) and \(t_2\) are the temperatures of each side of the material, \(A\) is the area of the slab, \(d\) is the thickness of the slab, and \(k\) is the thermal conductivity of the material.

Rearrange the formula to obtain an expression for \(t_2\)

Since \(Q = \frac{kA(t_1 - t_2)}{d}\) then \(Qd = kA(t_1 - t_2)\)

i.e. \(\frac{Qd}{kA} = t_1 - t_2\)

from which, \(t_2 = t_1 - \frac{Qd}{kA}\)

14. The slip, \(s\), of a vehicle is given by: \(s = \left(1 - \frac{r \omega}{v}\right) \times 100\%\) where \(r\) is the tyre radius, \(\omega\) is the angular velocity and \(v\) the velocity. Transpose to make \(r\) the subject of the formula.

Since \(s = \left(1 - \frac{r \omega}{v}\right) \times 100\%\) then \(\frac{s}{100} = 1 - \frac{r \omega}{v}\)

and \(\frac{r \omega}{v} = 1 - \frac{s}{100}\)

from which, \(r = \frac{v}{\omega} \left(1 - \frac{s}{100}\right)\)
15. The critical load, \( F \) newtons, of a steel column may be determined from the formula

\[
L \sqrt{\frac{F}{EI}} = n\pi \quad \text{where } L \text{ is the length, } EI \text{ is the flexural rigidity, and } n \text{ is a positive integer.}
\]

Transpose for \( F \) and hence determine the value of \( F \) when \( n = 1 \), \( E = 0.25 \times 10^{12} \text{ N} / \text{m}^2 \), \( I = 6.92 \times 10^{-6} \text{ m}^4 \) and \( L = 1.12 \text{ m} \)

Since \( L \sqrt{\frac{F}{EI}} = n\pi \) then \( \sqrt{\frac{F}{EI}} = \frac{n\pi}{L} \)

and \( \frac{F}{EI} = \left(\frac{n\pi}{L}\right)^2 \)

i.e. \( F = EI \left(\frac{n\pi}{L}\right)^2 \)

When \( n = 1 \), \( E = 0.25 \times 10^{12} \text{ N} / \text{m}^2 \), \( I = 6.92 \times 10^{-6} \text{ m}^4 \) and \( L = 1.12 \text{ m} \),

\[
\text{load, } F = EI \left(\frac{n\pi}{L}\right)^2 = \left(0.25 \times 10^{12}\right)\left(6.92 \times 10^{-6}\right)\left(\frac{1\times\pi}{1.12}\right)^2 = 13.61 \times 10^6 \text{ N} = 13.61 \text{ MN}
\]

16. The flow of slurry along a pipe on a coal processing plant is given by: 

\[
V = \frac{\pi pr^4}{8\eta\ell}
\]

Transpose the equation for \( r \)

Since \( V = \frac{\pi pr^4}{8\eta\ell} \) then \( 8\eta\ell V = \pi pr^4 \)

and \( \frac{8\eta\ell V}{\pi p} = r^4 \)

from which, \( r = \sqrt[4]{\frac{8\eta\ell V}{\pi p}} \)